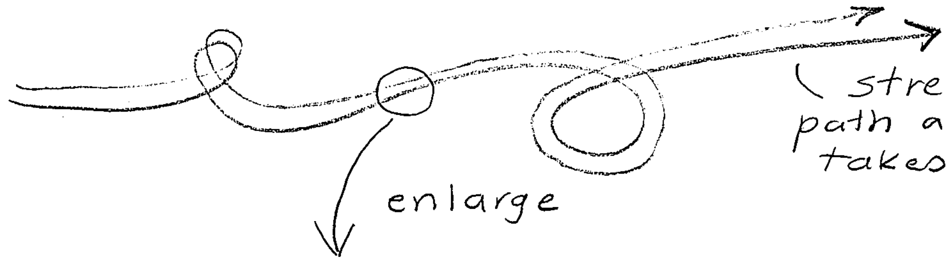
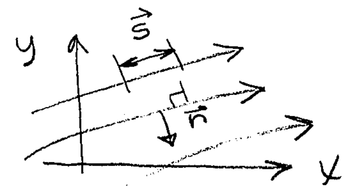
BERNOULLI EQN ALONG A STREAMLINE

1. COORDINATE SYSTEM





3 streamlines shown

$\vec{s}$ = unit vector <u>along</u> streamline

$\vec{n}$ = " " normal to "

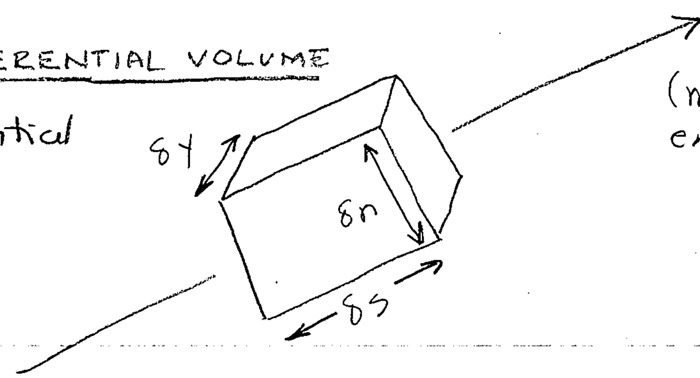
2. OBJECTIVE:

determine how elevation, velocity, and pressure vary <u>along</u> a streamline

3. FLUID DIFFERENTIAL VOLUME

$\delta\forall$ = differential volume

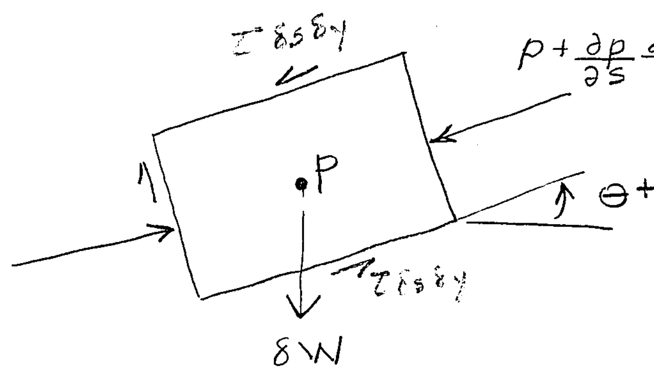
$\delta\forall = \delta y\,\delta n\,\delta s$



(much enlarged)

4. F.B.D.

let p = pressure in center of $\delta\forall$



Assume fluid is <u>inviscid</u> : $\Rightarrow \tau = 0$

5. Newton's 2nd law:

$$\Sigma \delta F_s = \delta m a_s \quad \text{where} \qquad (1)$$

$$\Sigma \delta F_s = \text{sum of differential forces}$$
$$= \left[\left(p - \frac{\partial p}{\partial s}\frac{\delta s}{2}\right) - \left(p + \frac{\partial p}{\partial s}\frac{\delta s}{2}\right)\right]\delta y \delta n - \delta W \sin\theta$$
$$= \left[-\frac{\partial p}{\partial s}\frac{\delta s}{2} - \frac{\partial p}{\partial s}\frac{\delta s}{2}\right]\delta y \delta n - \delta W \sin\theta$$

but $\delta W = \gamma \delta \forall = \gamma \delta s \delta y \delta n$

so $\Sigma \delta F_s = \left(-\frac{\partial p}{\partial s}\right)\delta y \delta s \delta n - \gamma \delta y \delta s \delta n \sin\theta$

now $\delta m = \rho \delta \forall = \rho \delta y \delta s \delta n$

a_s = along path

$$= \left.\frac{dV}{dt}\right|_s = \frac{\partial V}{\partial s}\frac{ds}{dt}$$

$$= \frac{\partial V}{\partial s} V$$

so (1) becomes

$$\left(-\frac{\partial p}{\partial s}\right)\delta y \delta s \delta n - \gamma \delta y \delta s \delta n \sin\theta = (\rho \delta y \delta s \delta n)\left(V\frac{\partial V}{\partial s}\right)$$

or
$$-\frac{\partial p}{\partial s} - \gamma \sin\theta = \rho V \frac{\partial V}{\partial s} \qquad (2)$$

if $\vec{a}_s = 0$, then (2) becomes $p = \gamma h$!

INTERMEDIATE USES OF (2)

3.1

3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

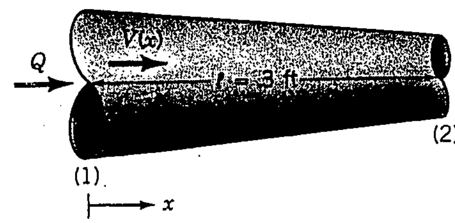


FIGURE P3.1

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} \quad \text{or} \quad \frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{\text{slugs}}{\text{ft}^3}\left(10\,\frac{\text{ft}}{\text{s}}\right)^2 (1+x)$, with x in feet

$$= \underline{\underline{-194(1+x)\,\frac{\text{lb}}{\text{ft}^3}}}$$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50\,\text{psi}}^{p_2} dp = -194 \int_{x_1=0}^{x_2=3} (1+x)\,dx$

or $p_2 = 50\,\text{psi} - 194\left(3 + \frac{3^2}{2}\right)\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,\text{psi}}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10\,\frac{\text{ft}}{\text{s}}$

$V_2 = 10(1+3) = 40\,\frac{\text{ft}}{\text{s}}$

Thus,

$$p_2 = 50\,\text{psi} + \frac{1}{2}\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)(10^2 - 40^2)\frac{\text{ft}^2}{\text{s}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = \underline{\underline{39.9\,\text{psi}}}$$

6. INTEGRATE (2):

- NOTE THAT $\delta s \cdot \sin\theta = \delta z$

$$\Rightarrow \sin\theta = \frac{\delta z}{\delta s} = \frac{dz}{ds}$$

- NOTE THAT $V\frac{\partial V}{\partial s} \equiv V\frac{dV}{ds}$ because we're considering along a streamline, not any other direction

- NOTE THAT $V\frac{dV}{ds} = \frac{1}{2}\frac{dV^2}{ds}$

- NOTE THAT $dp = \frac{\partial p}{\partial s}\,ds$

$\Rightarrow \frac{dp}{ds} = \frac{\partial p}{\partial s}$ if V does not change with time

$$\Rightarrow -\frac{\partial p}{\partial s} - \gamma\sin\theta = \rho V\frac{\partial V}{\partial s} \quad \text{becomes}$$

$$-\frac{dp}{ds} - \gamma\frac{dz}{ds} = \frac{1}{2}\rho\frac{dV^2}{ds}$$

or $$dp + \frac{1}{2}\rho\, d(V^2) + \gamma\, dz = 0$$

or $$\frac{dp}{\rho} + \frac{1}{2}d(V^2) + g\,dz = 0$$

$$\int\frac{dp}{\rho} + \frac{1}{2}\int d(V^2) + g\int dz = 0$$

ASSUME ρ IS A CONSTANT

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{CONSTANT} \quad \text{OR}$$

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant along a streamline}$$