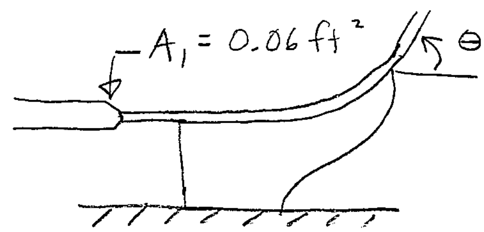
Example 5.5/132

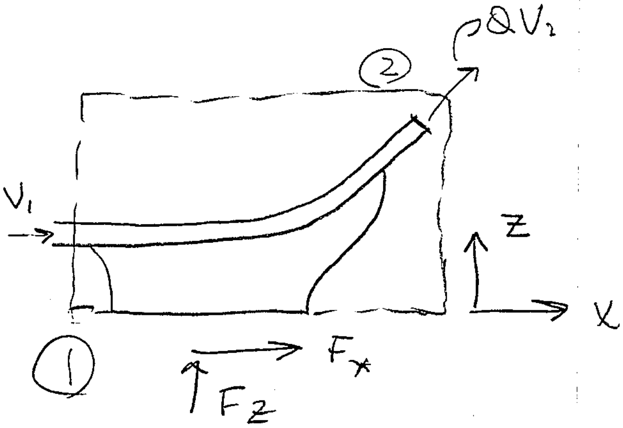


__GIVEN:__ Vane turns water at angle θ. Neglect weight of water and vane
$V_1 = 10\,ft/s$

__FIND:__ Anchoring force to hold vane

__SOLUTION:__

1. DRAW
   C.V., C.S., number $\rho Q V_1$
   a. since we want force to hold vane in place, draw around entire thing



   b. axes: $\longrightarrow x$ $\uparrow z$
   a. label c.s.
      1. incoming = 1
      2. outgoing = 2
   c. F.B.D.
      1. no pressure force
      2. no weight
      3. only reaction: F_x, F_z
         1. always assume positive - helps with signs
      4. "momentum" force
2. write full equation $\Sigma \vec{F} = \frac{\partial}{\partial t}\int \rho \vec{V} d\forall + \int \rho \vec{V}\, \vec{V}\cdot\vec{n}\, dA$
3. assumptions
   a. steady flow
   b. $\vec{V}\cdot\vec{n} = V$
   c. V uniform across c.s.
   d. ρ constant

4. equation: $\Sigma F = \rho Q(\vec{V}_2 - \vec{V}_1)$

$$\Sigma F_x = \rho Q(V_{2x} - V_{1x})$$
$$\Sigma F_z = \rho Q(V_{2z} - V_{1z})$$

5. solve

x-direction: $F_x = \rho Q(V_{2x} - V_{1x})$

because neglect gravity, $V_2 = V_1$

$V_{2x} = V_1 \cos\theta$

$$F_x = \rho Q(V_1 \cos\theta - V_1)$$
$$= \rho QV(\cos\theta - 1)$$
$$= -\rho QV(1 - \cos\theta)$$

z-direction: $F_z = \rho Q(V_{2z} - V_{1z})$

$V_{1z} = 0$

$V_{2z} = V\sin\theta$

$$F_z = \rho Q(V\sin\theta)$$
$$= \rho QV\sin\theta$$

$\rho = 1.94$ slugs/m³ (Table 1.4), cover

$Q = VA = \left(10\frac{ft}{s}\right)(0.06\ ft^2) = 0.6\ \frac{ft^3}{s}$

$$F_x = \left(-1.94\frac{slugs}{ft^3}\right)\left(0.6\frac{ft^3}{s}\right)(10 ft/s)(1-\cos\theta)$$

$$= -11.64(1-\cos\theta)\ \frac{slugs}{\cancel{ft^3}}\cdot\frac{\cancel{ft^3}}{s}\cdot\frac{ft}{s}$$

$$= -11.64(1-\cos\theta)\ lb$$

$$F_z = 1.94\frac{slugs}{ft^3}(0.6\ ft^3/s)(10 ft/s)(\sin\theta)$$

$$= 11.64\sin\theta\ lb$$

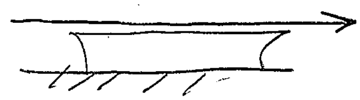
6. summary:

$$F_x = -11.64\ (1-\cos\theta)\ \text{lb}$$

$$F_z = 11.64\ \sin\theta\ \text{lb}$$

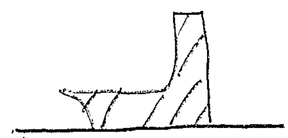
Cases

$\theta = 0$



$F_x = 0$

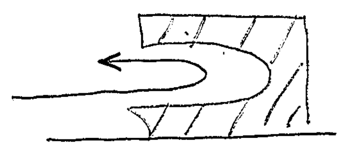
$F_z = 0$



$\theta = 90°$

$F_x = -11.64\ \text{lb}$

$F_z = 11.64\ \text{lb}$



$\theta = 180°$

$F_x = -11.64\ (1-(-1))$

$-23.28\ \text{lb}$

$F_z = 0$

see Figure E5.5d