Lecture 23

OBJECTIVE: Similitude and Modeling

TEXT: 7.1, 7.6, 7.8 - 7.9

NEXT: 8.1, 8.2

HW:

I. REVIEW QUESTIONS

II. DIFFICULTY IN FINDING SOLUTIONS - TURBULENCE & LOSSES

A. CHAOS THEORY - WHAT IS IT?

B. TURBULENT FLOW DIFFICULT

1. GALACTIC SCALE - SPIRAL-TYPE GALAXIES
2. MICROSCOPIC SCALE - DIFFUSION OF O_2 THROUGH CELL WALLS

C. SOLUTIONS (OR APPROXIMATIONS) ARE DEMANDED

1. PIPELINE SYSTEMS
2. DAMS, RESERVOIRS
3. SPACE SHUTTLE, AIRPLANES

D. WE DON'T KNOW ALL - HOW DO WE DO IT?

1. MODELING
2. USU EXAMPLES

III. SIMILARITY

A. PREVIEW QUESTION 4: 3 KINDS

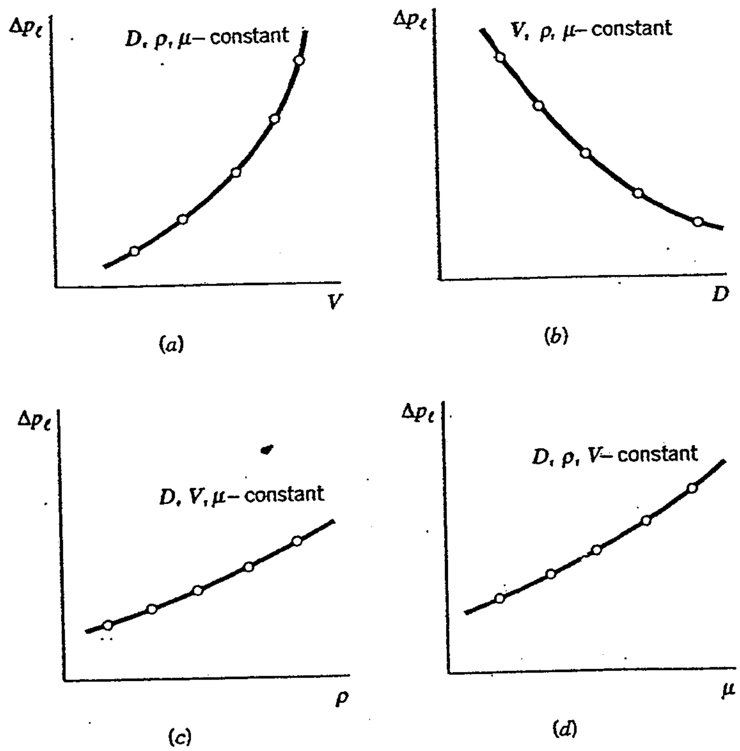
1. GEOMETRIC
2. KINEMATIC
3. DYNAMIC

B. SCALES - GEOMETRIC

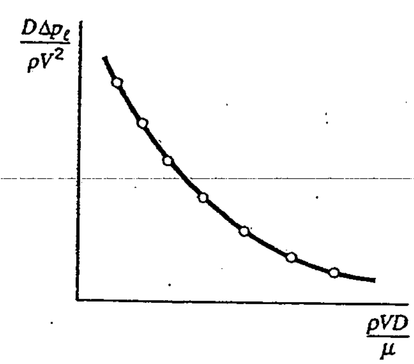
1. MODEL : PROTOTYPE

$$\frac{\text{MODEL}}{\text{PROTOTYPE}}$$

2. $\lambda_L = \text{SCALE} = \dfrac{L_m}{L_p}$



■ **FIGURE 7.1** **Illustrative plots showing how the pressure drop in a pipe may be affected by several different factors.**



■ **FIGURE 7.2** **An illustrative plot of pressure drop data using dimensionless parameters.**

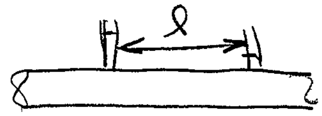
C. KINEMATIC: magnitude and direction of velocities

D. DYNAMIC: forces scale appropriately

IV. HOW TO ENSURE?

A. LIST VARIABLES OF IMPORTANCE

$$\frac{\Delta p}{l} = \phi(D, \rho, \mu, V)$$



B. FORM DIMENSIONLESS GROUPS : Preview # 2

$$\frac{D\,\Delta p}{\rho V^2 l} = \phi\left(\frac{\rho V D}{\mu}\right) = \phi(\Pi_1)$$

what you want

dimensionless group

C. Common ratios · preview question 3

D. make $\Pi_{i_m} = \Pi_{i_p}$

e.g. $Re_m = Re_p$

E. ALSO REDUCES EXPERIMENTS FROM 5^4 (3125) to 5^2 (25)

F. SUMMARY: preview question 4

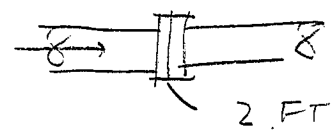
V. MODELING - COMMON MODELS

IF	THEN
totally surrounded by fluid	$\mathbb{R}e_m = \mathbb{R}e_p$
there is an interface (mostly liquid/air)	$\mathbb{F}r_m = \mathbb{F}r_p$

A. $\mathbb{R}e$ EXAMPLE 7.5

GIVEN: $Q_p = 30$ cfs, $D_{VALVE} = 2$ ft
WATER

FIND: Q_m IF $D = 3$ in.



SOLUTION:

1. $\lambda_L = \dfrac{L_m}{L_p} = \dfrac{3\text{ in} \times \frac{1\text{ ft}}{12\text{ in}}}{2\text{ ft}} = \dfrac{0.25\text{ ft}}{2\text{ ft}} = 0.125 = \dfrac{1}{8}$

2. $\mathbb{R}e_m = \mathbb{R}e_p$ OR $\dfrac{V_m L_m}{\nu_m} = \dfrac{V_p L_p}{\nu_p}$

e.g. $\dfrac{V_m}{V_p} = \dfrac{\nu_m}{\nu_p}\dfrac{L_p}{L_m} = \dfrac{8}{1}$ where L refers to D

To get Q, recall $Q \doteq \underbrace{V \cdot L^2}$ looking at dimensions

so $\dfrac{V_m L_m}{\nu_m} \cdot \dfrac{L_m}{L_m} = \dfrac{V_p L_p}{\nu_p} \cdot \dfrac{L_p}{L_p}$

$$\frac{Q_m}{\nu_m L_m} = \frac{Q_p}{\nu_p L_p}$$

or $\dfrac{Q_m}{Q_p} = \dfrac{\nu_m L_m}{\nu_p L_p} = \dfrac{1}{8} \Rightarrow Q_m = \dfrac{Q_p}{8} = 3.75$ cfs

B. $\mathbb{F}r$ EXAMPLE 7.7

GIVEN: $L_p = 20m$, $Q_p = 125 m^3/s$
$\lambda_L = 1:15$, $T_p = 24 hr$

FIND: L_m, Q_m, T_m

SOLUTION:

$$\mathbb{F}r_m = \mathbb{F}r_p \quad \text{or} \quad \frac{V_m}{\sqrt{g L_m}} = \frac{V_p}{\sqrt{g L_p}}$$

or in more basic dimensions,

$$\frac{L_m T^{-1}{}_m}{\sqrt{g L_m}} = \frac{L_p T^{-1}{}_p}{\sqrt{g L_p}}$$

$$\Rightarrow \frac{L_m}{T_m \sqrt{g L_m}} = \frac{L_p}{T_p \sqrt{g L_p}} \quad \text{or} \quad \frac{T_m}{T_p} = \lambda_T =$$

$$\frac{\sqrt{L_p}}{L_p} \cdot \frac{L_m}{\sqrt{L_m}} = \frac{L_m^{1/2}}{L_p^{1/2}} = \lambda_L^{1/2} = \lambda_T$$

e.g. $T_m = \lambda_L^{1/2} T_p = \left(\frac{1}{15}\right)^{1/2} 24 hr = 6.20 hr.$

FOR Q, $$\frac{V_m}{\sqrt{g L_m}} \frac{L_m^2}{L_m^2} = \frac{V_p}{\sqrt{g L_p}} \frac{L_p^2}{L_p^2}$$

$$\frac{Q_m}{Q_p} = \frac{L_m^{5/2}}{L_p^{5/2}} = \lambda_L^{5/2} \quad \text{or}$$

$$Q_m = \lambda_L^{5/2} Q_p = \left(\frac{1}{15}\right)^{5/2} (125 m^3/s) = 0.143 m^3/s$$

$$L_m = \frac{1}{15} L_p = \frac{20}{15} = 1.33 m$$

To Find Scales of Interest

1. Select Reynolds or Froude number

2. Equate the model Pi parameter to the prototype Pi parameter

3. Write out the full meaning of the Pi parameter

$$Fr = \frac{V}{\sqrt{gL}}, \qquad \mathrm{Re} = \frac{Vl}{\nu}$$

3. List the scales that you need, for example,

   λ_V, λ_F, λ_T

4. Inspect the Pi parameters to see if the necessary scale is already there