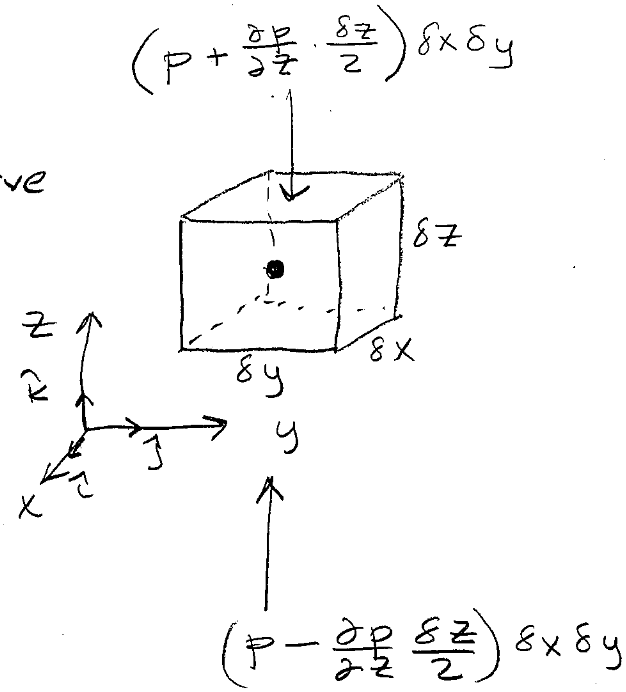
IV. PRESSURE VARIATION IN VERTICAL

$\delta \equiv$ "delta"

$\partial \equiv$ partial derivative



A. consider elemental volume $\delta x\,\delta y\,\delta z$

B. F.B.D. for vertical direction

$$\sum \delta\vec{F} = \delta m\vec{a} = 0$$

so $0 = -$weight $+$ pressure force

$$\text{weight} = -\gamma\,\delta\forall\,\hat{k} = -\gamma\,(\delta x\,\delta y\,\delta z)\,\hat{k}$$

$$\text{pressure force} = \left[\left(p - \frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\,\delta y - \left(p + \frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\,\delta y\right]\hat{k}$$

$$\Rightarrow \delta\vec{F} = -\gamma\,\cancel{\delta x}\,\cancel{\delta y}\,\cancel{\delta z}\,\hat{k} - \frac{\partial p}{\partial z}\,\cancel{\delta x}\,\cancel{\delta y}\,\cancel{\delta z}\,\hat{k} = 0$$

or $\dfrac{\partial p}{\partial z} = -\gamma$ but since $\dfrac{\partial p}{\partial x} = \dfrac{\partial p}{\partial y} = 0,$

$$\frac{dp}{dz} = -\gamma$$

integrate: 1. separate variables

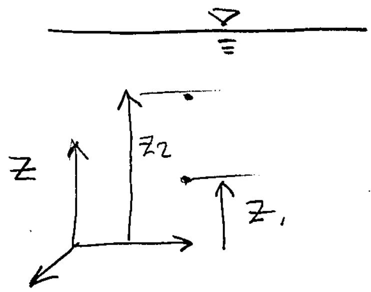
$$\int dp = -\int \gamma\,dz$$

2. assume ρ is constant

$$\int dp = -\gamma \int dz$$

3. assign definite variables

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$



4 integrate

$$p_2 - p_1 = -\gamma (z_2 - z_1)$$

5. let $h = z_2 - z_1$

$$p_2 - p_1 = -\gamma h$$

6. find pressure at point 1 (lower elevation)

$$p_1 = p_2 + \gamma h$$

7. take point 2 at interface where pressure = atmospheric, or $p_{gage} = 0$

$$p_1 = \gamma h$$